Introduction into

Multi-Factor Investing

By Yury Polyakov

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Abstract

Asset managers worldwide face the new market reality: significantly decreased global economic growth, the lowest interest rates and inflation, an increase in asset cross-correlation due to accelerated globalization and spikes of event driven volatility, high assets valuation levels.

In response to market changes the new approaches to asset management have been developed: risk based asset allocation and factor investing. In this introductory document we briefly cover historical evolution of asset management from traditional methods to alternative beta and “pure” alpha strategies. Then we describe each category and analyze it based on our own examples. At the end we outline the new trends and future fields of research in modern asset management and alpha generating strategies.

Table of content

I. Introduction

1. What global portfolio investors are looking for from their asset managers?

- High equity like return.
- Bond like low volatility.
- Avoidance of large drawdowns (recessionary peak to trough drop in assets value)
- Diversification from their currently owned assets (own business or real estate for individuals, current investment exposure for institutional investors).

In other words investors in general seek the greatest possible efficiency in terms of return/risk ratio where the risk is uncertainty of investment outcomes.

2. New market reality:

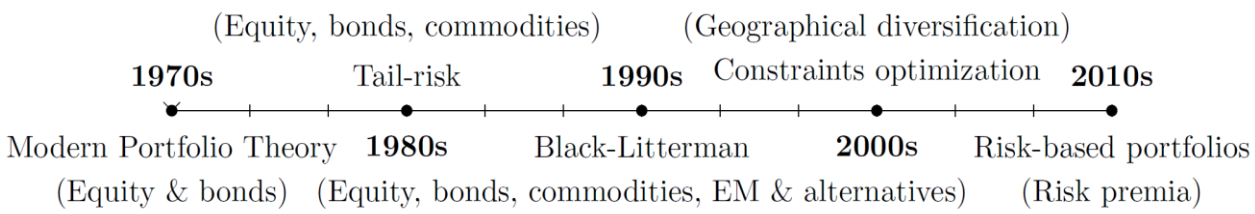
- Decreasing global economic growth.
- Interest rates and inflation are at the lowest levels.
- Risk premiums depressed across all major traditional asset classes due to high valuation levels.
- Speed of information distribution and globalization make event driving volatility much bigger.
- Diversification potential is reduced: traditional asset classes are limited in number and became highly cross-correlated due to accelerated globalization.
- Growth risk (equity beta) still dominates investment portfolios worldwide.

These factors make traditional asset allocation along with popular nowadays risk parity portfolios not effective anymore. Estimated 10 year looking forward Sharpe ratios for these portfolios are well below historical mean values for the last thirty years (see page 14).

3. Adapting to new conditions: historical evolution of asset management

The figure below shows historical evolution of asset management techniques from traditional approach which is based on Modern Portfolio Theory (MPT) and Efficient Market Hypothesis (EMH) to factor based investing. The concept of a risk factor stems from the belief that the returns on an asset can be broken down or split up into distinct sub-components that each contribute to the overall return and risk characteristics of the asset. Extending this into portfolio management, Podkaminer (2013) likened risk factors to atoms and assets to molecules. For example, the return earned on a corporate bond can be broken down according to the risks to which the bond holder is exposed, including duration, inflation and credit risks.

Figure 1. Historical evolution of asset management



Source: Jacob Buhl Jensen 2013

Factor based approach gives the following advantages (among others):

- An increased ability to understand, predict and explain the drivers of performance over different time horizons and regimes.
- Factors pairwise correlation is much lower than traditional asset classes even during turbulent times, robust portfolio mean-variance optimization is possible[1]
- Gives an ability to unwind from undesired risk exposure and consistently exploit various forms of market inefficiency (alternative beta).

However "pure" factor investing has several practical impediments. The main problem is effective factor capturing and its investability given investment policy determined constraints and liquidity requirements. Most popular nowadays smart beta indexes have unlevered long only exposure. That leads to residual undesired risk exposure (traditional beta), reduce diversification and factor's performance potential.

4. **We divide asset management approaches into the three main categories:**
    1. Traditional asset management.
    2. Factor based investing.
    3. "Pure" alpha strategies.

    The third category is "pure" alpha consistently derived by several top hedge funds like Renaissance Technologies, Bridgewater Pure Alpha, Two Sigma, AQR Capital, Millennium (they closed for new investors long time ago and very costly for old clients - fee structure 3/30+) from the following sources:
    1. Idiosyncratic risk exposure: security selection based on superior information or analytical skill
    2. Market and factor timing models (volatility, return and correlation forecasts)
    3. Portfolio adaptive dynamic allocation (optimization techniques)
    4. Event-driven strategies, arbitrage and ultra high frequency trading
    5. Value creation (top private equity funds or activist strategies in public markets)

    The main idea of active management is to determine and combine as many independent alpha sources as possible in order to generate high and stable investment results uncorrelated to traditional assets.

5. **The future of active factor based asset management lies in the following fields:**[2]
    - Determined global investable factor universe database that includes popular traditional, alternative and not widely known pure alpha generating factors.
    - Effective factor exposure via ranking approach which is superior to popular alternative indexes weighting methods.
    - Robust factor timing models.
    - Robust portfolio optimization schemes.
    - Goal seeking theme investing (theme is a combination of factors with a special purpose).
    - Diversified absolute return pure alpha multi-strategy portfolios at low cost.

[1] See Podkaminer research 2013, Milliman research 2015

[2] See the full Advantex presentation available upon request, NDA may be required

II. Traditional asset management based on Modern Portfolio Theory and Efficient Market Hypothesis.

Theory.

EMH states that it is impossible to "beat the market" because the market efficiency causes existing asset prices to always incorporate and reflect all relevant information. Under this paradigm security selection or market timing is useless.

MPT states that investors can construct an "efficient frontier" of optimal portfolios offering the maximum possible expected return for every given level of risk. The maximum return can only be achieved through the optimal combination of asset classes rather than individual securities. The ideal combination of asset classes for every level of risk can be found through what is known as mean variance optimization (MVO), which creates the optimal mix based on each asset class's expected return, expected volatility and expected correlation.

Investment universe and return drivers.

Underlying investment universe for such approach is cap weighted indexes of different asset classes: equity, fixed income, commodities, real estate, currencies or cash equivalents (money market instruments). Risk and return drivers of such assets are fundamental risk factors and associated risk premiums (returns) also named as traditional beta risk factors. The most important fundamental factors are economic growth, interest rates and inflation that drive the major portion of the returns across all asset classes. Broad asset class indexes capture many other individual risk factors as well which contribute to overall index performance in different directions and almost eliminate or diversified away each other in an index combination.

Figure 2. Investment universe: traditional asset management building blocks

Source: Charles Schwab Investment Advisory, Inc.

The need of diversification.

Asset classes perform differently in different macroeconomic conditions and market regimes. For example, when economic growth and interest rates are raising equity related assets perform significantly better than fixed income assets. And vice versa when economic growth is slow and interest rates are falling bonds outperform equity on risk adjusted basis (in terms of Sharpe ratio). In the long run different asset classes normalized to required level of volatility (using leverage or deleverage) have more or less the same return (Sharpe ratio is close to each other, see Bridgewater research). Therefore due to obvious inability of an average investor or an asset manager reliably forecast future economy and market conditions the diversification across different asset classes is needed.

Since the returns are not perfectly correlated, losses of any one asset tend to be offset by gains on other assets. In this manner, the risk of a portfolio may well be less than the average risk of its constituent assets[3]. Nevertheless the great problem of the traditional asset management is that the number of asset classes are limited and cross-correlations between them are high thereby significantly decreasing the power of diversification.

Asset allocation and portfolio construction methods in traditional asset management.

A. Investment policy determined asset allocation.

Small variation in asset allocation within specified range is allowed. Commonly used by policy constrained mutual, pension, sovereign funds, endowments. Passive investments contrarian by nature (when an asset grew in value above threshold you sell it and vice versa which is contrarian to trend following or momentum strategy). That allocation is easy to implement, has low transaction costs due to low portfolio turnover and very high liquidity. As a result such type of investments is available for any investors at the lowest management fees – typically below 1% annually.

Advantages:

Simplicity; low management and transaction costs; very high liquidity; no need of leverage and short positions; doesn't require estimates for assets return, volatility and correlations for optimization; tax efficient.

Disadvantages:

Not actually diversified investment exposure; risks significantly shifted towards growth related factors (equity beta); sizable drawdowns and low investment efficacy measured by return/risk ratio; policy constraints lead to sub optimal portfolio allocation.

Policy determined traditional portfolio example:

1. Assets

We used the follwing asset (ETFs) for traditional portfolio construction that cover the major portion of traditional risk premia sources:

[3] It is widely known from math that volatility of equally weighted portfolio of zero correlated equal volatility assets is reduced by N^0.5 in comparison to individual asset's volatility, where N is the number of uncorrelated assets.

1. TLT – 20+ year US government bonds
2. BND – broad investment grade bond index
3. BNDX – broad investment grade bond index ex. US
4. SPY – S&P 500 index – large cap equity
5. IWM – Russell 2000 – small cap equity
6. VWO – international equity index
7. IYR – US real estate
8. IAU – gold index

All ETFs are replicated before inception dates by Portolfio123.com.

2. Simulation assumptions:

- Strategy: policy determined capital allocation
- Underlying assets: traditional cap weighted indexes (ETFs).
- Tested period:31 Dec 1998 - 26 Feb 2016, daily data provided by P123.com
- Dividends reinvested
- Transaction costs: 0.25% one way transaction costs for rebalancing (daily)
- Risk free interest rate: 3M Libor
- Cost of leverage (if used): 3M Libor + 1.5% margin

3. Performance description

Figure 3 shows capital and risk allocation for such a portfolio. One can see that relatively balanced capital allocation leads to unbalanced risk allocation, calculated as the average assets volatility contribution to overall portfolio volatility. Fixed income assets have negative risk contribution due to negative correlation to growth related assets (equity beta).

Figure 4 shows portfolio equity curve vs SPY (dividends reinvested) and equity drawdowns graph (peak to trough drop in value). In 2008 SPY lost more than 50% in value while strategy equity lost about 30%. At the same time portfolio correlation to SPY is 0.89, that means portfolio risks significantly shifted towards equity beta. Sharpe ratio is 0.52 versus 0.22 for SPY assuming 3M Libor as a risk free rate of return.

Table 1. Traditional static portfolio performance

Results (annualized on daily data)		
Parameter	**Portfolio**	**SPY**
Mean return	7.42%	6.39%
Standard deviation	10.46%	19.91%
Sharpe	0.52	0.22
Drawdown	31%	55%
Turnover	0%	0%
Leverage max	1.00	1.00
Leverage min	1.00	1.00
Correlation to SPY	0.89	1.00
Beta	0.47	1.00
Max one year realized volatility	35%	47%
Min one year realized volatility	4%	9%
End value (100 at beginning)	327	214

Figure 3. Traditional portfolio capital and risk allocation.

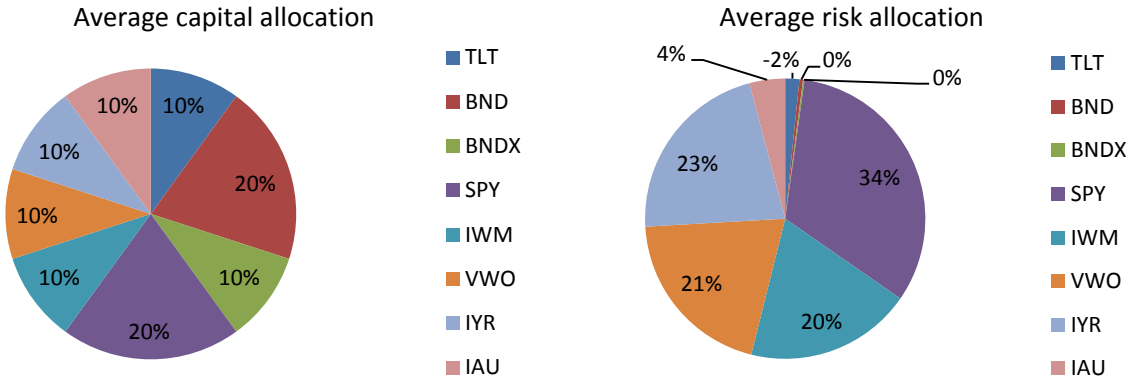
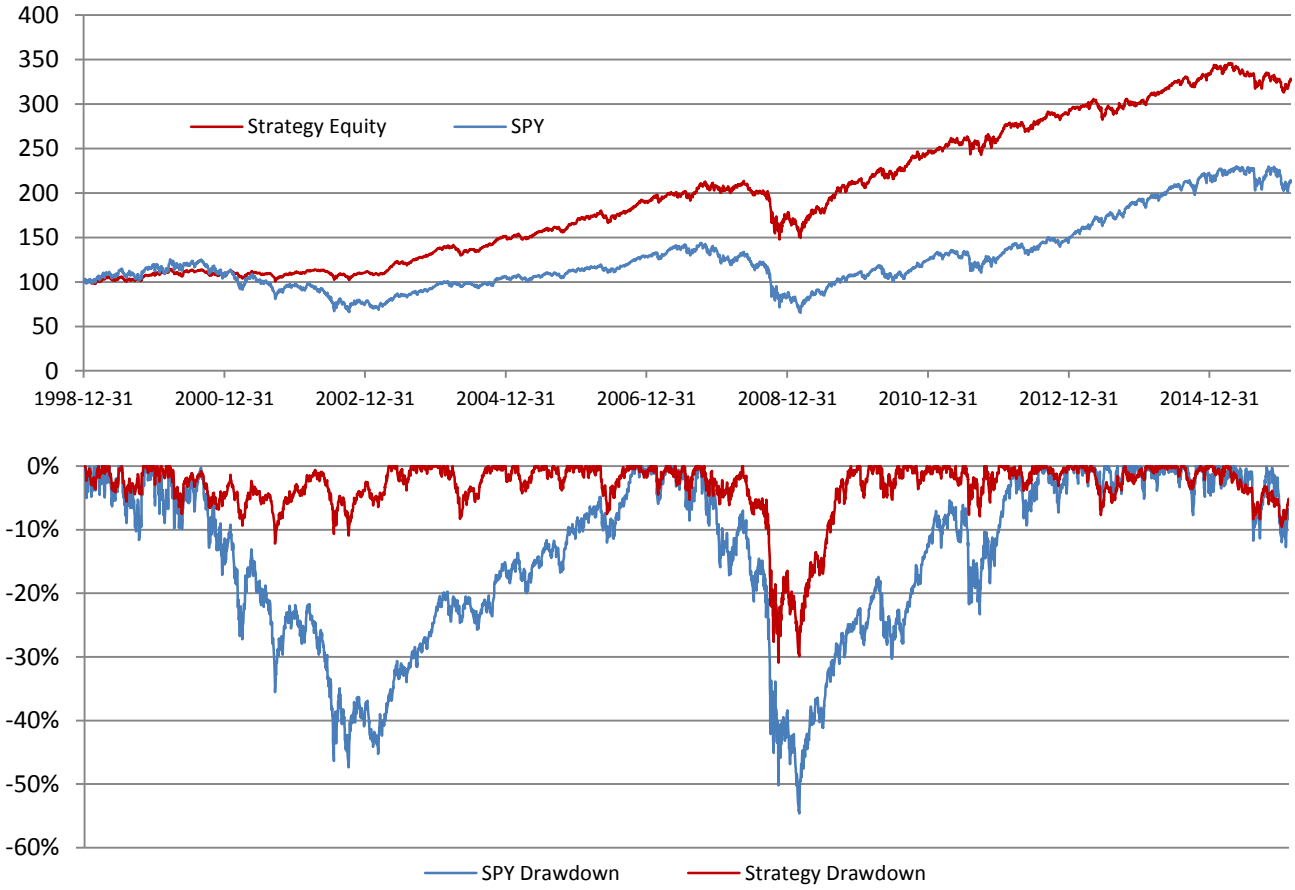


Figure 4. Traditional portfolio performance vs SPY ($100 is a base)



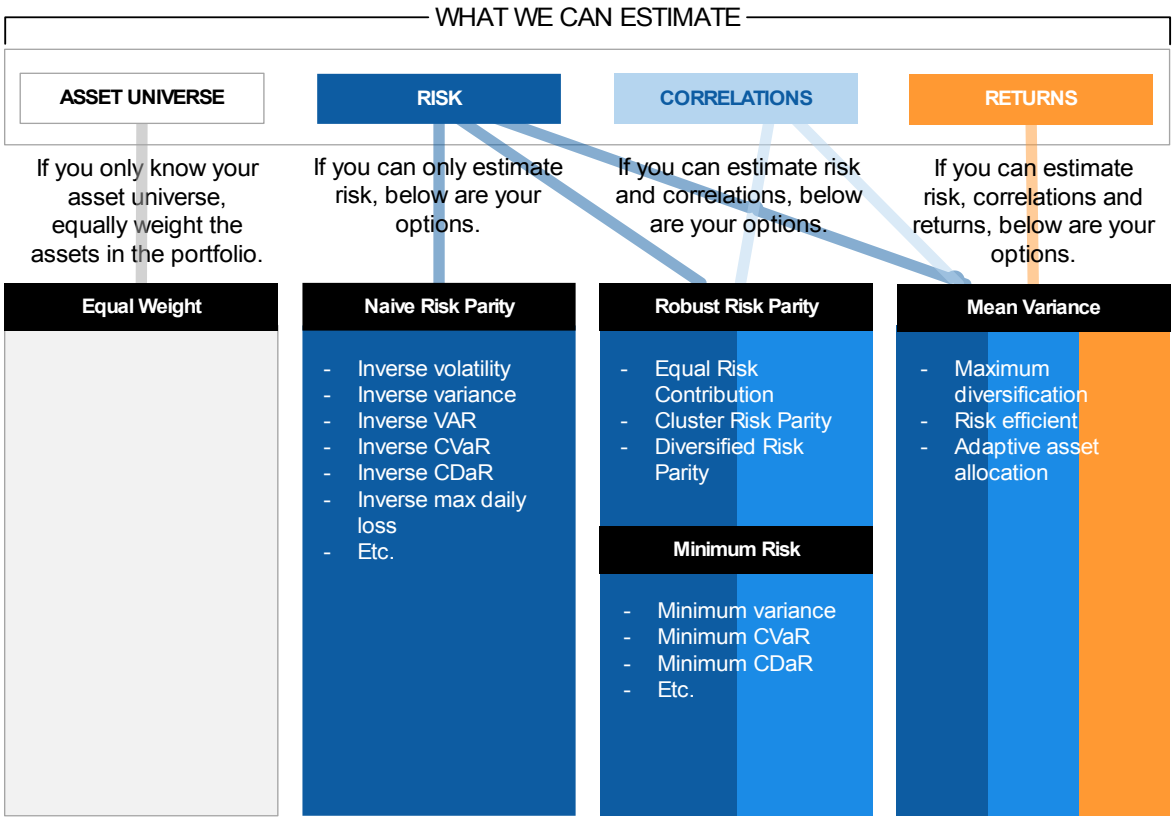
Return	1999	2000	2001	2002	2003	2004	2005	2006	2007
Portfolio	10.01%	-0.25%	0.65%	0.65%	20.37%	12.08%	9.00%	14.47%	8.86%
SPY	19.56%	-7.56%	-10.13%	-20.73%	26.11%	10.77%	5.14%	15.12%	6.28%

Return	2008	2009	2010	2011	2012	2013	2014	2015	2016
Portfolio	-13.11%	19.45%	15.84%	6.40%	10.95%	4.05%	10.13%	-1.85%	0.35%
SPY	-37.33%	26.97%	15.59%	4.27%	15.40%	28.55%	13.18%	2.44%	-3.35%

B. Risk based allocation – various risk focused forms of mean variance optimization.

Used by hedge funds and private wealth funds. Mean variance optimization (MVO) requires volatility, return and correlation estimates. Full MVO which maximize portfolio Sharpe ratio is unstable due to input errors particularly return estimates. Small change in inputs gives large change in output asset weights making such allocation unreliable and unstable. Therefore in practice a special case of MVO - robust risk parity allocation method is used (also named equal risk contribution or ERC) and became very popular nowadays (Ray Dalio's Bridgewater All Weather Fund is the first fund that follows this strategy) which takes into account only covariance matrix (volatility and correlations that are more stable over time).

Figure 5. Mean variance optimization methods



Our backtest with the same assets as in the first example shows significant increase (~1.5 times after all transaction and leverage costs) in Sharpe ratio even if only volatility estimates is used. That allocation is named inverse volatility or naïve risk parity allocation. The key features are high leverage (in order to get required level of portfolio volatility), higher transaction costs, capital allocation significantly shifted towards low volatility assets like government or corporate investment grade bonds.

Advantages:

No need of estimates of assets return, diversified investment exposure, more balanced return profile – no large drawdowns, as a result higher return/risk ratio in comparison to policy determined allocation.

Disadvantages:

Requires high leverage to get targeted level of volatility and return, requires estimates for asset volatilities and correlation, higher portfolio turnover, lower liquidity, as a

result higher transaction and management costs. Capital allocation (not risk) significantly shifted towards low volatility assets such as investment grade bonds. Return estimates are out of consideration.

Figure 6. Inverse volatility allocation

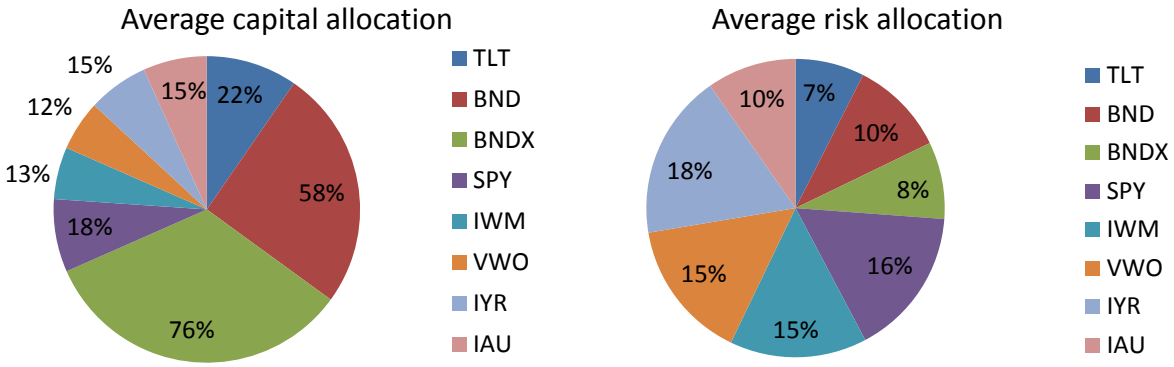
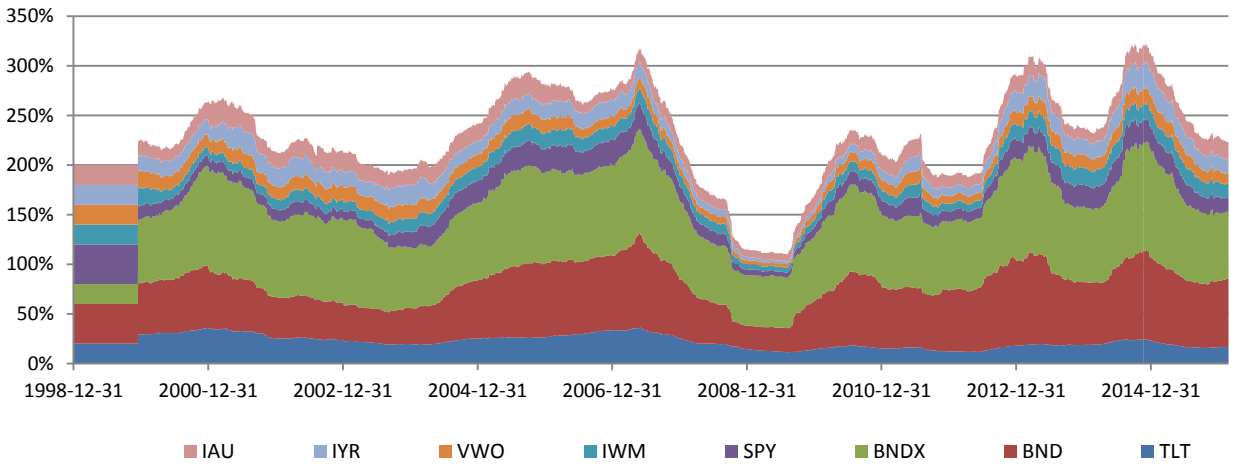


Figure 7. Capital allocation



Portfolio description and performance.

As can be seen from *figure 6* inverse volatility allocation portfolio has more balanced risk structure while capital allocation significantly shifted towards low volatility fixed income assets.

For comparison purposes on *figure 9* we added popular risk parity benchmark - Salient index (green line). The Salient Risk Parity Index is a quantitatively driven global asset allocation index that seeks to weight risk equally across four asset classes — equities, rates, commodities and credit.

Figure 8. Salient Risk Parity Index allocation

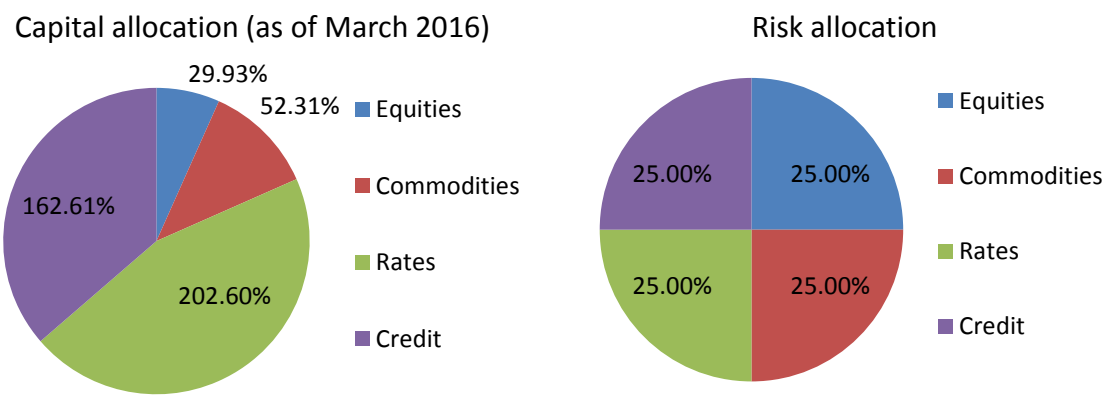
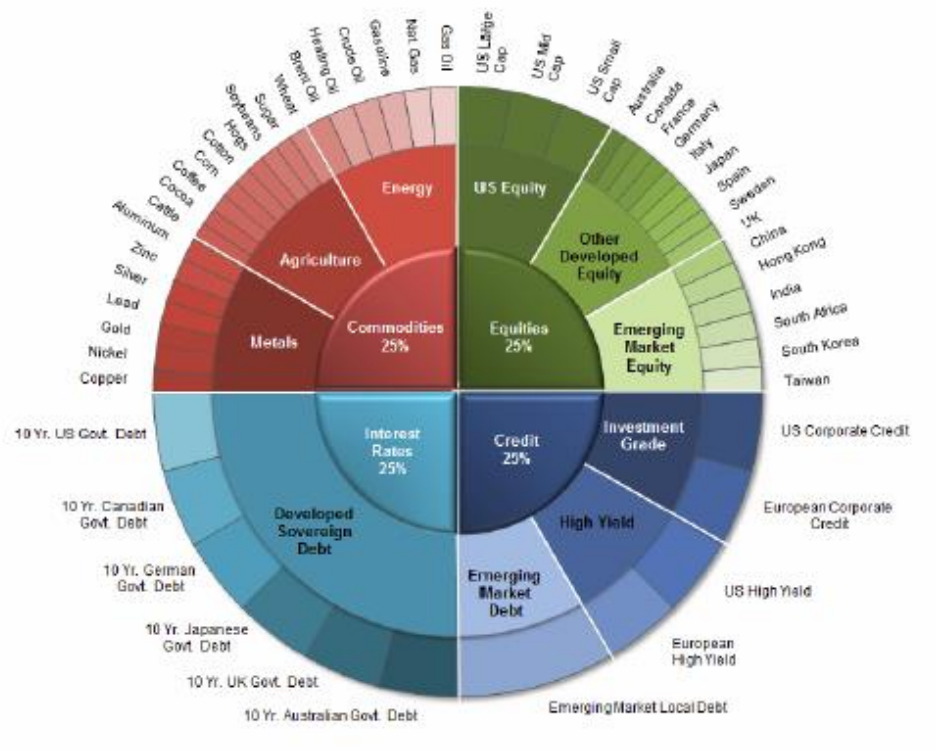
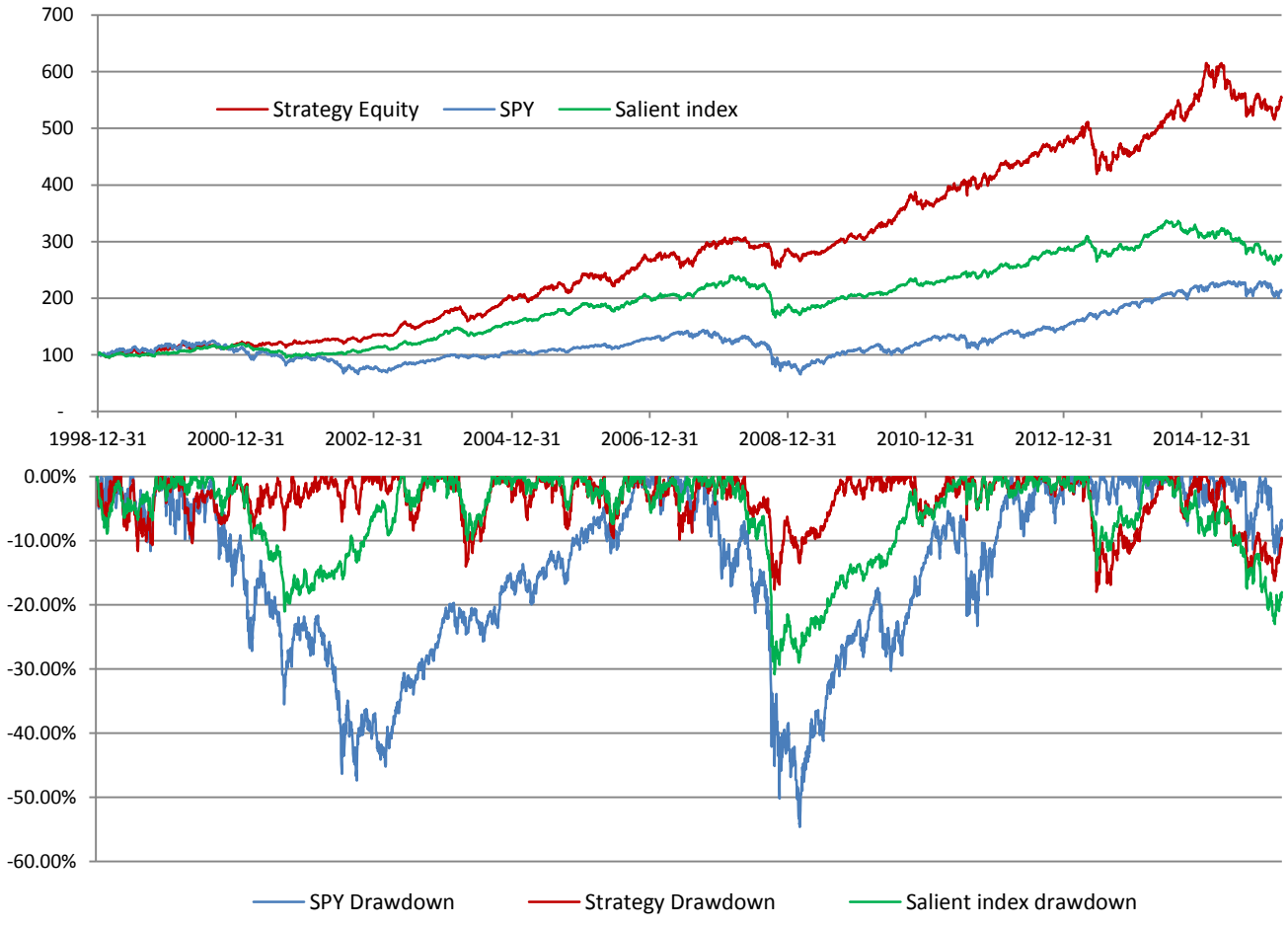


Figure 8. Salient Risk Parity Index asset allocation



Source: www.salientindices.com

Figure 9. IV Portfolio equity curve and drawdowns vs SPY and Salient index.



Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
Portfolio	13.27%	5.03%	3.27%	11.23%	23.59%	18.07%	13.15%	15.51%	11.60%
SPY	19.56%	-7.56%	-10.13%	-20.73%	26.11%	10.77%	5.14%	15.12%	6.28%
Salient Index	4.92%	11.24%	-15.47%	11.99%	19.75%	15.24%	16.45%	8.67%	11.24%

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Portfolio	-2.48%	6.28%	19.98%	11.95%	13.40%	-2.59%	21.67%	-4.80%	3.30%
SPY	-37.33%	26.97%	15.59%	4.27%	15.40%	28.55%	13.18%	2.44%	-3.35%
Salient Index	-17.42%	9.38%	11.42%	8.74%	14.36%	0.69%	8.17%	-12.22%	0.94%

Table 2. Traditional inverse volatility portfolio performance[4]

Results (annualized on daily data)			
Parameter	**Strategy**	**SPY**	**Salient index**
Mean return	10.51%	6.39%	6.26%
St dev	10.90%	19.91%	8.67%
Sharpe	0.78	0.22	0.49
Max drawdown	18.02%	54.63%	30.84%
Turnover	197%	0.00%	N/A
Leverage max	3.23	1.00	N/A
Leverage min	1.10	1.00	N/A
Correlation to SPY	0.57	1.00	0.42
Beta	0.31	1.00	0.18
Max one year realized volatility	19%	47%	N/A
Min one year realized volatility	6%	9%	N/A
End value (100 at beginning)	554	214	276

Inverse volatility portfolio has only 18% max drawdown and Sharpe ratio close to 0.8. However such portfolio requires frequent rebalancing and high leverage: portfolio turnover (calculated as change in levered asset positions on absolute basis to equity) is almost 200%, the leverage reaches 3.23 when assets volatility is low (in real trading index futures can be used with embedded leverage instead of example ETFs). Portfolio correlation to SPY is 0.57 while equity beta is quite low due to lower portfolio standard deviation comparing to SPY. The average portfolio assets cross-correlation is 0.14 during the tested period (see *table 3*).

Anyway capital based allocation as in the first example or risk focused allocation suffer from the same problem – current global market environment that significantly limits potential future performance of traditional risk premiums irrespective to allocation schemes.

Table 3. Traditional assets correlation table

Correlation	TLT	BND	BNDX	SPY	IWM	VWO	IYR	IAU
TLT	1.00	0.71	0.64	(0.36)	(0.36)	(0.33)	(0.22)	0.08
BND	0.71	1.00	0.56	(0.18)	(0.22)	(0.11)	(0.11)	0.11
BNDX	0.64	0.56	1.00	(0.23)	(0.24)	(0.24)	(0.12)	0.09
SPY	(0.36)	(0.18)	(0.23)	1.00	0.88	0.69	0.72	0.01
IWM	(0.36)	(0.22)	(0.24)	0.88	1.00	0.67	0.73	0.02
VWO	(0.33)	(0.11)	(0.24)	0.69	0.67	1.00	0.61	0.14
IYR	(0.22)	(0.11)	(0.12)	0.72	0.73	0.61	1.00	0.02
IAU	0.08	0.11	0.09	0.01	0.02	0.14	0.02	1.00

[4] For portfolio construction, we used one year rolling (250 trading days) past volatility.

C. Traditional risk premia global outlook

1. Growth in developed and emerging markets is under pressure. Projected future growth is significantly lower than in previous decades including emerging markets as well.

Figure 10. Global growth headwinds

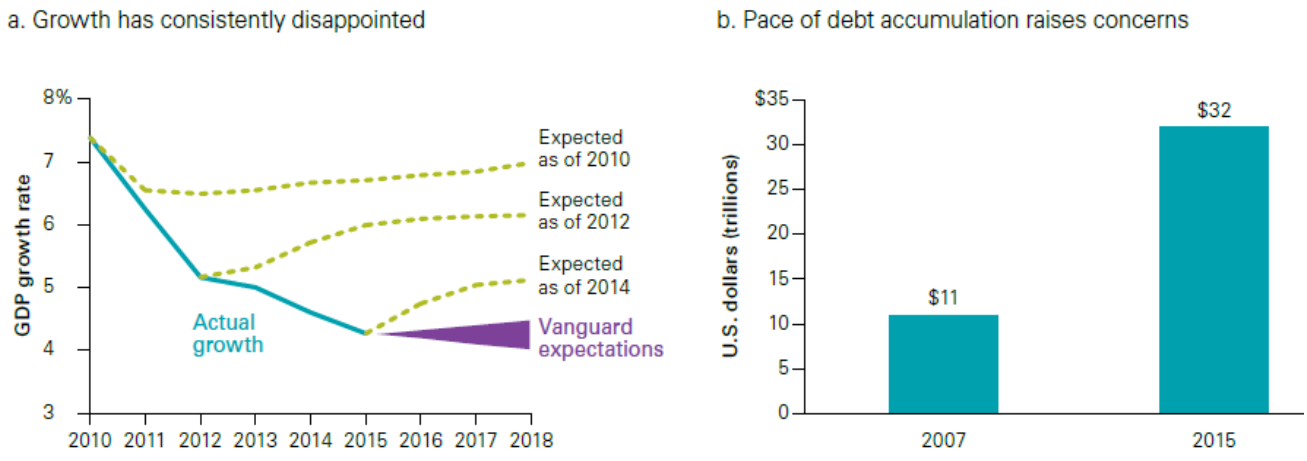
	United States	Euro area	China	Japan	United Kingdom	Canada	Australia
Percentage of world GDP	22.4%	17.1%	13.3%	6.2%	3.7%	2.3%	1.9%
Estimated trend growth rates (%)							
Pre-recession average (1990–2007)	3.0	2.0	10.0	1.4	2.9	2.5	3.4
Projected future (2016–2020)	2.1	1.5	6.3	0.5	2.1	2.0	2.8
Growth headwinds							
Slowing growth of labor force *Slower population growth and aging of population*							
Private-sector debt deleveraging *Debt-deleveraging cycle, constraining willingness to spend*							
Sluggish capital investment *Falling cost of technology and demographic effects on businesses' growth plans*							
Fiscal sustainability and committed fiscal austerity *Unsustainable debt dynamics may result in suboptimal policies and uncertainty*							
Commodity exports dependency *Weak commodity price outlook*							
Currency strength *Tighter financial conditions, weaker manufacturing and exports*							
Rising income inequality *Falling purchasing power of consumers with highest propensity to spend*							

Highly significant factor Moderately significant factor Factor not present

Notes: Slowing growth of labor force: Birth rates minus mortality rates (slope of the trend line, 1960–present); **Private-sector debt deleveraging:** Percentage increase in household debt (% of GDP) from 2008 to December 2015; **Sluggish capital investment:** Difference between average fixed capital formation as percentage of GDP, 2000–2007 and 2008–latest; **Fiscal sustainability and committed fiscal austerity:** Fiscal space estimates based on Moody's Economy.com model, as of February 2015 and difference in structural government budget balance over next two years (2016–2017); **Commodity exports dependency:** Qualitative assessment of commodity export dependence; **Currency strength:** Level of real effective exchange rate as of September 2015 (>100, overvalued/<100, undervalued); **Rising income inequality:** Average percentage point change in the income share of top 1% of income (1980–2010). Also, for China, we factor local government debt into our debt deleveraging rating.

Sources: Vanguard calculations, based on data from International Monetary Fund (IMF)—World Economic Outlook, Organisation for Economic Co-operation and Development, United Nations, U.S. Bureau of Economic Analysis, U.S. Federal Reserve System, Moody's Analytics, and Thomson Reuters Datastream.

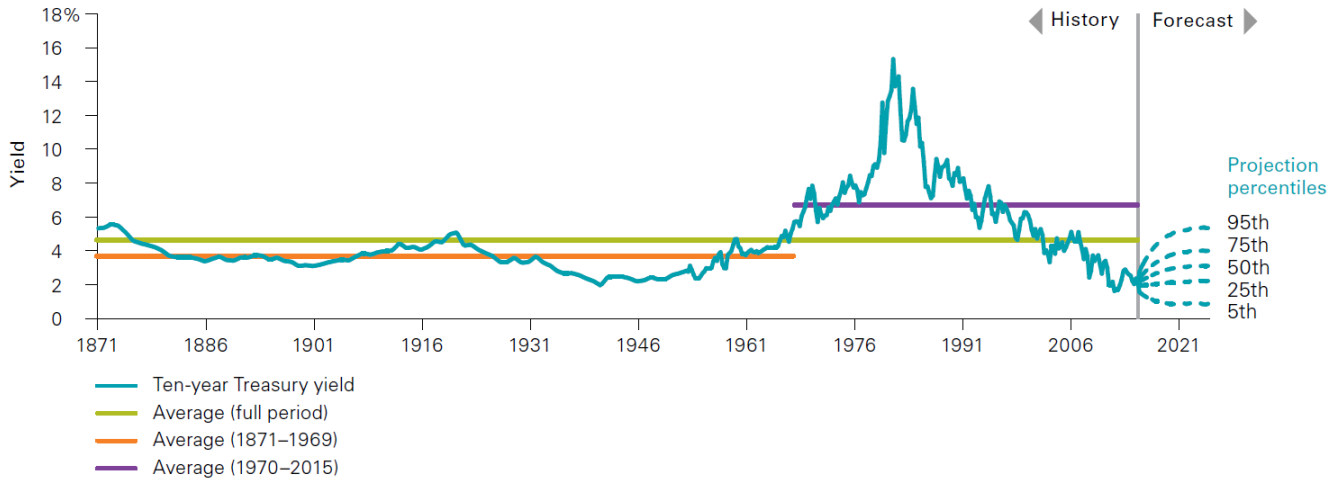
Figure 11. Emerging market growth expectations



Sources: Vanguard calculations, based on data from IMF, Organisation for Economic Co-operation and Development, and J.P.Morgan.

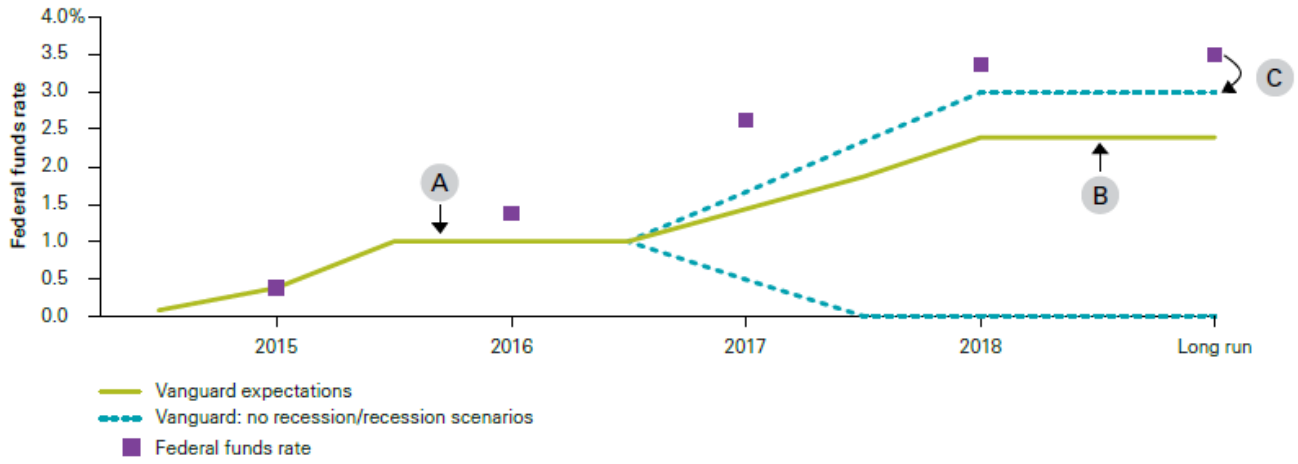
2. Lowest interest rates and inflation put traditional risk parity allocation into question.

Figure 12. Interest rates levels



Note: 10-year Treasury yield projections based on 10,000 simulations from VCMM as of September 2015.
Sources: Vanguard calculations, based on data from Robert Shiller website, at aida.wss.yale.edu/~shiller/data.htm.

Figure 13. Interest rates forecast



A. Likely extended pause at 1%:
- Removes "exigent circumstances."
- Further tightening risks additional USD strength and deeper manufacturing recession.
- Allows tapering of balance-sheet reinvestment.
- Pause ensures negative real FFR.

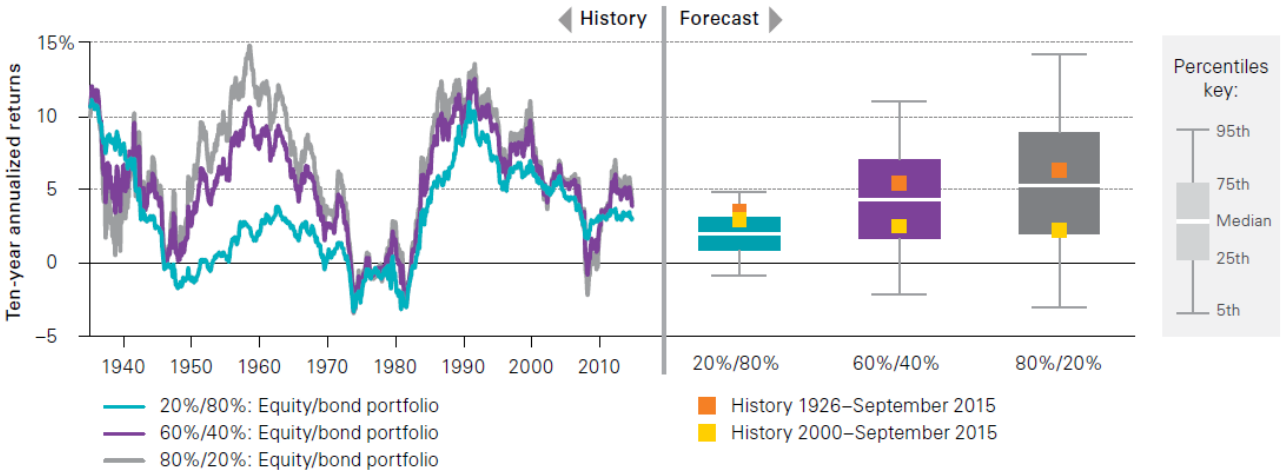
B. Average projection:
Weighted average of 0% and 3%, assuming 20% probability of recession at some point.

C. Fed's long-term dots still too high:
- Vanguard's view of structural deceleration is consistent with 3% nominal FFR (1% real FFR).
- Historical average of 4% biased upward by 1980–2000 period of high real FFR needed to anchor inflation expectations.

Notes: Purple dots represent median expectation at stated year-end from Federal Reserve Board's September 2015 *Summary of Economic Projections*. Green line represents Vanguard's estimate of appropriate Fed policy adjusted for probability of recession in any one year beginning in 2018. Blue dashed lines represent binary outcomes of recession (line approaching and remaining at zero) or no recession (line approaching and remaining at 3%). FFR = federal funds rate.
Sources: Vanguard calculations, based on data from Federal Reserve Board.

3. Lower projected Sharpe ratio in future for both equity centric and fixed income tilted traditional portfolios.

Figure 14. Projected ten year return outlook for balanced portfolios



Equity/bond portfolios	Bottom 5th percentile	25th percentile	50th percentile	75th percentile	95th percentile	History 1926–2015	History 2000–2015
20%/80%	–0.8%	0.8%	1.9%	3.1%	4.8%	3.5%	2.9%
60%/40%	–2.1%	1.8%	4.3%	7.0%	10.9%	5.5%	2.6%
80%/20%	–3.0%	2.0%	5.3%	8.9%	14.2%	6.2%	2.2%

Notes: Forecast displays 5th/25th/50th/75th/95th percentile ranges of 10,000 VCMM simulations for projected ten-year annualized real returns as of September 2015 in USD. Historical returns are computed using indexes defined in "Indexes used in our historical calculations," on page 5. The equity portion of the portfolios is 60% U.S. equity and 40% global ex-U.S. equity. The bond portion of the portfolios is 70% U.S. bonds and 30% global ex-U.S. bonds.

Source: Vanguard.

Summary of traditional asset management

Key Highlights:

- Underlying instruments: cap weighted indexes.
- Forecasting models: static exposure or market timing models.
- Asset allocation model: typically naïve risk parity asset allocation (MVO is not robust due to input estimation errors and high assets cross-correlation).
- Policy constraints: leverage, shorting, derivatives, liquidity, capital capacity.

Advantages:

- Simplicity (for static allocation).
- Low cost, opened for new investors.
- Large liquidity and invested capital capacity.

Disadvantages:

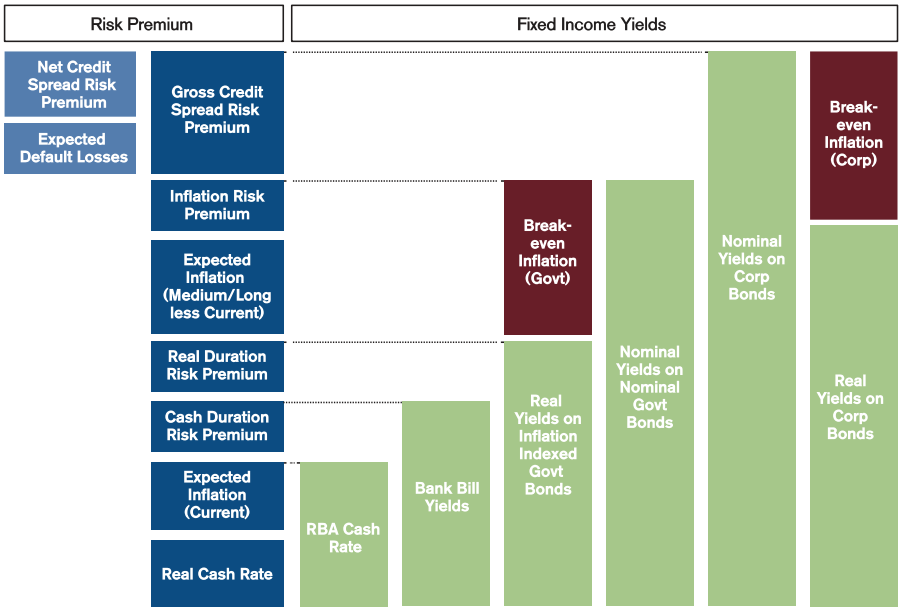
- High asset class cross-correlation, as a result lack of diversification and high volatility. Not normal return distribution: fat left tails and as a result large drawdowns.
- Overall portfolio risk typically significantly shifted towards equity risk premium.
- Not robust mean-variance optimization.
- Policy constraints lead to sub optimal portfolio allocation.
- As a result low Sharpe ratio especially looking forward.

III. Factor based investing.

Description

The concept of a risk factor stems from the belief that the returns on an asset can be broken down or split up into distinct sub-components that each contribute to the overall return and risk characteristics of the asset. Extending this into portfolio management, Podkaminer (2013) likened risk factors to atoms and assets to molecules. For example, the return earned on a corporate bond can be broken down according to the risks to which the bond holder is exposed, including duration, inflation and credit risks.

Figure 15. Risk factors that drive fixed income returns



Source: Milliman and Innova Asset Management

We define risk factor is a casual driver of asset returns which has risk, return and relationship characteristics with other risk factors. All risk factors have a degree of uncertainty or risk associated with them, and a corresponding risk premium which may be positive or negative.

We divide risk factors universe into three categories:

1. Traditional factors. Positive risk premia exists in the long run and market equilibrium. That factors do not contradict to modern portfolio theory or market efficacy hypothesis.
2. Aternative beta. Typically based on various forms of market inefficiencies raised from behavioral biases, investors constraints and barriers. Positive risk premium may or may not exist in market equilibrium and typically has cyclical nature. Also named as smart, exotic and style beta.
3. Idiosyncratic risk factors. Unique characteristics of specified securities.

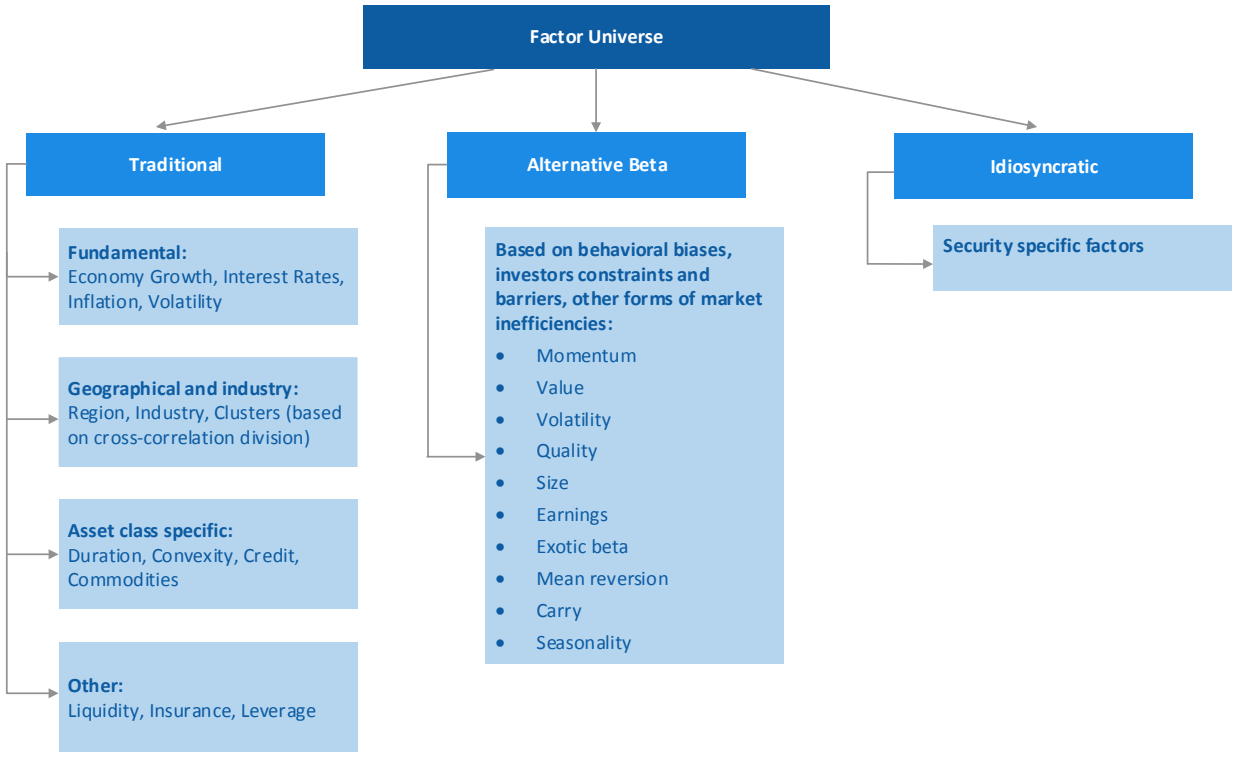
Geometrical interpretation

Geometrically this concept looks like the following. If we imagine actually multidimensional risk factor exposure – risk premia (predicted return) space in usual to us three dimensions (where X and Y axis are factors and Z is return) we see a surface that looks like a sea. There are waves and cavities of risk premia. At the bottom of the

wave there are long term macro-economy factors which comprise broad market exposure, at the middle are alternative beta factors and at the top pure alpha idiosyncratic factors. All that components are moving in the course of time (which actually is another axis into factor space) and prone to stochastic component movement as well.

As a result assets risk and return can be expressed as a function of factors. For example, share return is a linear function of broad equity market risk factor which comprises many macro-economy sub-factors (traditional equity beta) + alternative beta factors + alpha or idiosyncratic risk factors + stochastic component.

Figure 16. Factors decomposition



An understanding of factor exposures provides investors with the opportunity to move the focus of the allocation decision from asset classes to factor exposures. The factor-based investing framework thus attempts to identify and allocate to compensated factors—that is, to factors expected to earn a positive return premium over the long term. Research into the factor framework has flourished in recent years and has found that the approach has the potential to improve risk-adjusted returns when used in conjunction with a range of investment portfolio configurations.

Vanguard outlook

Despite the recent interest in factor-based investing, related concepts have existed for decades. For example, value investing, discussed in Graham and Dodd (1934), can be considered a type of factor-based investing. Rather than diversifying across the entire market, value investors focus on a subset of stocks with specific characteristics such as attractive absolute or relative valuations. As this approach gained in popularity, style indexes (both value and growth, for instance) were introduced to better measure the performance of style investors and to provide them with passive vehicles to replicate the returns of active investors. Whether through an index or active

management approach, style investing allows portfolios to be created with a style tilt, or, put another way, a factor exposure. Style investments were specifically designed to have return and risk characteristics that differ from those of the broad market.

Table 4 outlines seven commonly discussed factor exposures that are notable both for the extensive literature documenting each, and for the empirical evidence of historical positive risk-adjusted excess returns associated with them.

Table 4. Popular factors descriptions

	Description
Market	Stocks have earned a return above the risk-free rate.
Value	Inexpensive stocks have earned a return above expensive stocks.
Size	Stocks of small companies have earned a return above stocks of large companies.
Momentum	Stocks with strong recent performance have earned a return above stocks with weak recent performance.
Low volatility	Stocks with low volatility have earned higher risk-adjusted returns than stocks with high volatility.
Term	Long-maturity bonds have earned a return above short-maturity bonds.
Credit	Low-credit bonds have earned a return above high-credit bonds.

Source: Vanguard.

Factor exposures and returns

As the investment universe has grown beyond stocks and bonds, the drivers of investment returns have become less transparent. Although the relationship between a stock portfolio and the broad market, or a bond portfolio and interest rates, is often clear, it may not be the case for other more complex strategies. Often, it is not immediately obvious what affects the returns of investments such as those in an active portfolio, hedge fund, or alternatively weighted (smart-beta) index. As reported in a number of academic studies, however, factor exposures appear to influence the return of these sometimes complex investments. For example, Bhansali (2011) demonstrated that common factor exposures exist across a diverse range of investments. Research has also shown that the returns of various indexes can be explained by factor exposures. For instance, Amenc, Goltz, and Le Sourd (2009), Jun and Malkiel (2008), and Philips et al. (2011) found that the return on alternatively weighted indexes can be explained by factor exposures.

In addition to explaining returns on asset-class investments, research has demonstrated that excess returns generated by active managers can also be related to factor exposures. Bender et al. (2014) provided evidence that up to 80% of the alpha (excess return) generated by active managers can be explained by the factor exposures of their

portfolios. Similarly, Bosse, Wimmer, and Philips (2013) demonstrated that factor tilts have been a primary driver of active bondfund performance. Both studies showed not only that factors play a role in determining the returns of passive investments, but that they also appear to play a critical role in the returns of successful active managers.

Explaining factor returns.

Debate continues on the investment rationale supporting certain factor returns. In some cases—for example, the equity market factor—a strong economic rationale exists for an excess return premium. The equity market premium has been deeply researched, and, although there is uncertainty over the future size of the premium, it is widely accepted that over the long term a positive excess return (above the "risk-free" rate) will be associated with the equity market factor. For many other factors, however, both the logic and economics explaining potential return premiums are subject to debate.

Table 5 briefly summarizes the investing rationale supporting seven sample factors. There are two main schools of thought on the rationale behind factor returns—risk and investor behavior. Briefly, the risk explanation posits that return premiums are simply rewards for bearing risk or uncertainty. This explanation, consistent with rational asset pricing, assumes that investors obtain return premiums as a reward for being exposed to an undiversifiable risk. The unequivocal view of the equity market factor is that it earns investors a premium as a reward for bearing the uncertainty of the value of future cash flows. In contrast, the behavioral argument holds that certain factor returns are caused by investor behavior. That is, investors make systematic errors that result in distinct patterns in investment returns. Systematic errors, for example, have been offered as an explanation for the existence of the momentum effect. Although the return premiums of some factors have been shown to be clearly related to risk, debate over the source of returns for other factors is more contentious. Nonetheless, investors should be aware of the arguments surrounding specific factors, as this may shape whether and how they allocate to these factors.

Table 5. Popular factor's risk premia explanations

	Risk explanation Premiums are consistent with rational pricing	Behavioral explanation Premiums are a result of suboptimal investor behavior
Market	Economic uncertainty; borrowing constraints; uncertainty about long-run risks.	Loss aversion and concern over short-term volatility of wealth.
Value	Cyclical risk of positive correlation between economic activity and security's returns.	Recency bias leads to investors shunning distressed firms and overpaying for recent growth.
Size	Cyclical risk of smaller firms being more exposed to changing, negative economic activity and default risk.	NA
Momentum	NA	Underreaction to new information being incorporated in asset prices.
Low volatility	Leverage and institutional (benchmarking) constraints.	"Lottery" effects leading to preference for high-volatility stocks with small chance of large payouts.
Term	Inflation uncertainty; supply/demand factors.	Loss aversion at longer maturities; role of bonds as a safe-haven asset.
Credit	Default and downgrade risk; positive correlation to economic activity.	NA

Source: Vanguard.

Future return premiums

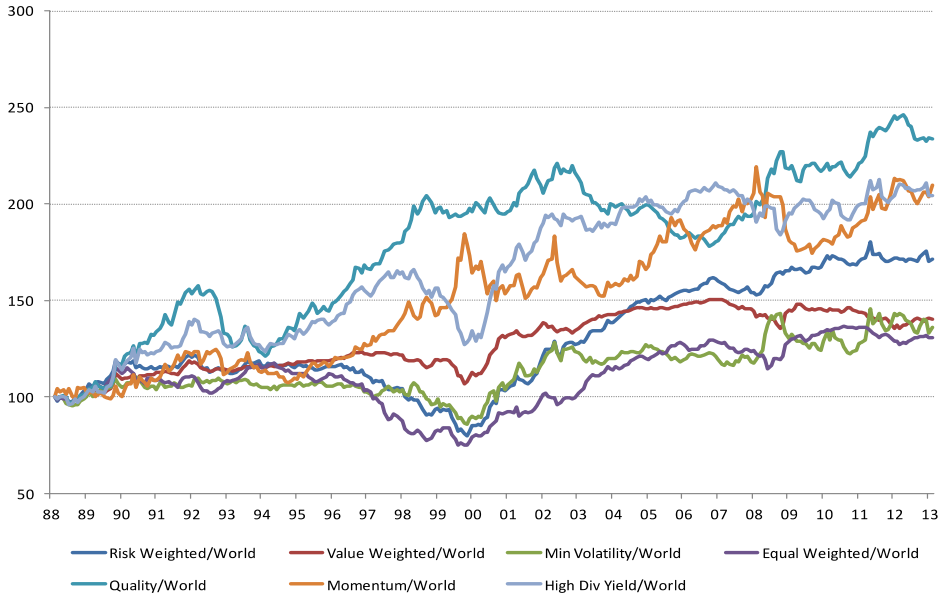
Expected returns are an important consideration for any investment. Although investors may already be familiar with a range of factor exposures and confident that those exposures will generate positive future returns over the long term, the future returns of other factor exposures may not be so clear. Indeed, there is some conjecture over whether the historical returns associated with certain factors will persist in the future. For example, Lo and MacKinlay (1990), Black (1993), and Harvey et al. (2014) contended that the empirical evidence is a result of datamining. As it stands, the debate is far from settled and continues in academia and industry. If the behavioral explanation holds for a factor, it may indicate a risk that the return premium may disappear if investors recognize their errors and modify their behavior accordingly—thus adding another layer of uncertainty to the future return premium. Investors may also fear that once a factor has been identified in the academic literature, it will be arbitraged away. Van Gelderen and Huij (2014) have argued against this, however, finding evidence that excess returns from factors are sustained even after they are published in the academic literature. Clearly, although investing in general is associated with a great deal of uncertainty, factor-based investing, of its own accord, has additional unique complexities that investors should consider when evaluating expected returns.

Return cyclicality

Similarly to asset-class returns, factor returns can be highly cyclical, and investors should be aware that individual factors may underperform for extended time periods. Although this risk is not unique to factor-based investing, it highlights the need for a long-term and disciplined perspective when assessing the factor-based investing framework. As we previously noted, empirical research on factors has found evidence that over the long term some factors have earned excess returns.

That research has also demonstrated that the same factors can underperform for lengthy periods. A key component in capturing any potential long-term premium is the investor's ability to stay the course during periods of poor performance.

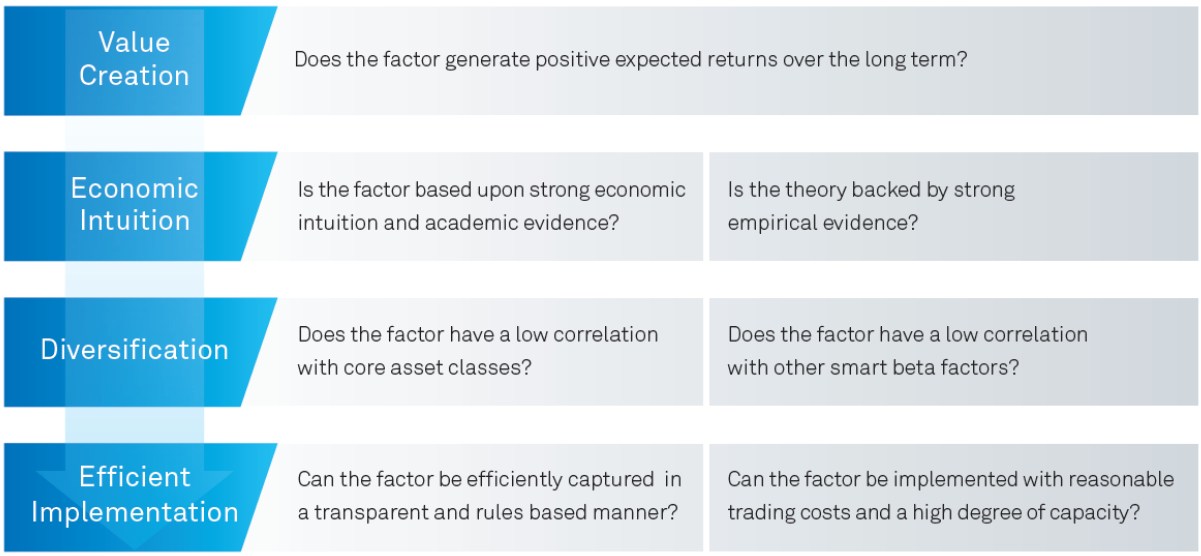
Figure 17. MSCI Barra Smart Beta performance relative to broad MSCI World index is cyclical



Practical implementation:

1. Factor universe identification.
In order to build multifactor portfolio investable factor universe should be determined. On *figure 18* the checklist for the factor inclusion presented. Once factor universe and underlying assets base are determined and each factor has its own quantitative descriptors we construct factor portfolios using ranking.

Figure18. Factor checklist below for the inclusion into factor universe.



Source: BlackRock Smart Beta, Feb 2015

2. Ranking.
The underlying principle is to rank the universe of underlying assets (stocks for example) based on their descriptors values. Descriptors are quantitative parameters of securities that have similar economic meaning and high correlation to each other, for example P/E, P/Sales, P/EBITDA, P/FCF for value factor, or beta, volatility, downside volatility for volatility factor. Select a subset of the constituents of the universe, for example top 5% for the long position, bottom 5% for the short. Then different weighting schemes applied. Weighting can be determined by market capitalization, inverse of volatility, inverse of variance or various other methodologies.

3. Filters, constraints and optimization.
Pure ranking method for portfolio construction gives mixed factor exposure because of undesired tilts towards different countries, industries or other factors as well. Therefore additional constraints and filters (market cap, liquidity, price etc) is used for factor portfolio construction. Optimization techniques can be used as well to catch securities cross-correlations, for example principal component cluster analysis for underlying asset universe. Long only portfolios capture major portion of traditional beta as well. Poor portfolio construction leads to significant shifts in factor exposure and lousy results.

4. Overall multifactor portfolio allocation.
Once factor portfolios are ready we combine them into one portfolio to achieve diversification effect. Weighting can be determined by risk focused allocation like risk

parity approach or can be based on full mean variance optimization using factor timing models[5].

5. Monitoring and rebalancing

Depending on chosen factor exposures and portfolio turnover rebalancing period is selected. On each rebalancing date current assets positions should be moved to theoretically derived portfolio positions taking into account transaction costs.

Multifactor portfolio example:

Underlying assets:

1. Fixed income:
   - TLT – 20+ year US government bonds
   - BND – broad investment grade bond index
   - BNDX – broad investment grade bond index ex. US
2. US euity long exposure:
   - Dynamic theme –mixed cap factor combination
   - Estimates – analyst estimates based system
   - Low volatility – volatility factor
   - Small cap – size factor
   - Momentum – momentum factor
   - Value – value factor
   - Quality – quality factor
3. US euity short exposure:
   - RWM – short Russell 2000 index
   - Smom – short leg momentum
   - Sval – short value
   - Squal – short quality

Simulation assumptions:

- Strategy: multi-factor portfolio
- Allocation: history based inverse volatility
- Stocks weihgtening scheme within factor portfolios: equally weighted
- Underlying assets: fixed income ETFs and US stocks
- Tested period:31 Dec 1998 - 26 Feb 2016, daily data provided by P123.com
- Dividends reinvested
- Transaction costs: 0.35% one way transaction costs for rebalancing (daily)
- Risk free interest rate: 3M Libor
- Cost of leverage: 3M Libor + 1.5% margin

Performance description

Figure 19 shows capital and risk allocation for such a portfolio. One can see that short systems reduce overall risk. Portfolio capital equally split between fixed income and equity based components.

[5] See the full presentation for further details

On *figure 19* we see capital allocation at any given moment of time. During low volatility periods the leverage required is close to 400%. During turbulent time the leverage goes down below 100% (it means deleverage – some portion of assets is into cash instruments that bring 3M Libor interest rate, for borrowing costs we used 3M Libor + 1.5%).

Table 6 shows portfolio performance stats vs SPY and Salient Index (dividends reinvested) and equity drawdowns. As can be seen balanced multifactor portfolio significantly outperformed traditional risk parity benchmark – Salient Index. Portfolio beta is only 0.11 and maximum drawdown is 16%. Portfolio average return is 15.54% at 9.23% volatility, Sharpe – 1.47. At the same time this strategy is sensitive to transactions and borrowing costs. In our simulation we have two layers of transaction costs: on microlevel within each factor portfolio and the second level is overall system allocation costs which equal to 0.35% per one way transaction. Portfolio turnover is over 600% (2.4% of equity capital on average on daily basis).

Figure 19. Multifactor portfolio

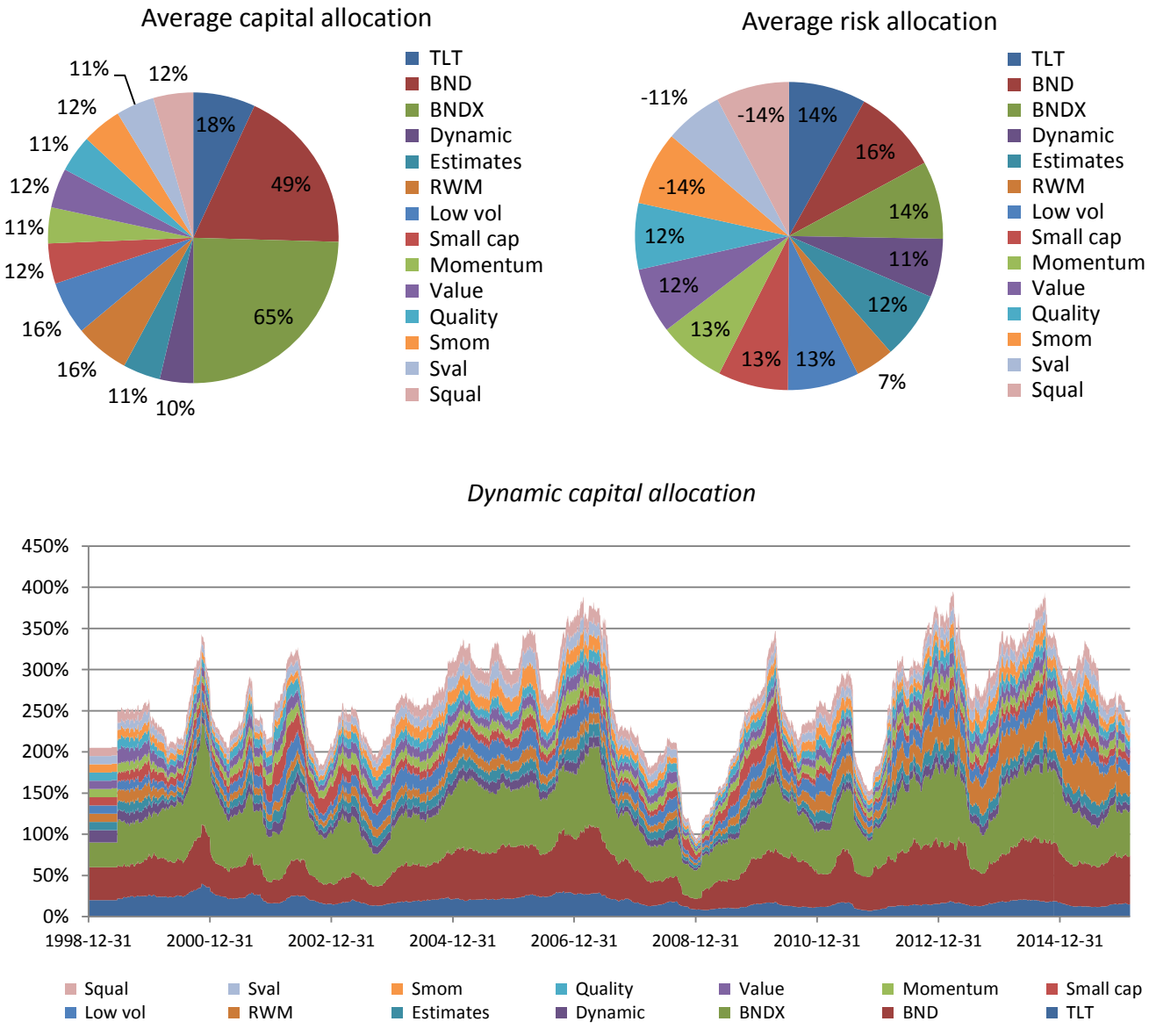
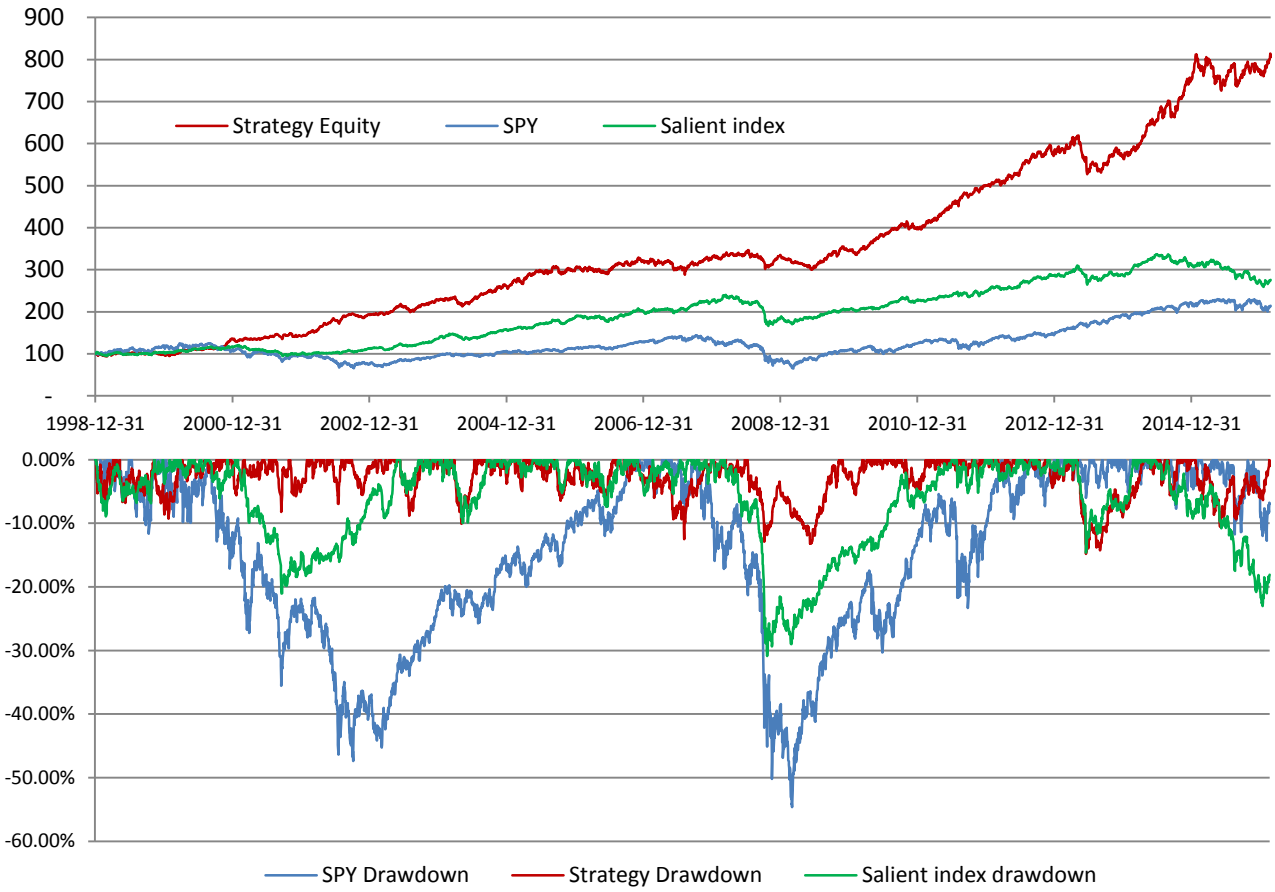


Figure 20. Multifactor portfolio equity curve and drawdowns graph



Annual return

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
Portfolio	-7.18%	46.60%	17.82%	42.07%	21.99%	15.75%	17.22%	7.56%	1.12%
SPY	19.56%	-7.56%	-10.13%	-20.73%	26.11%	10.77%	5.14%	15.12%	6.28%
Salient Index	4.92%	11.24%	-15.47%	11.99%	19.75%	15.24%	16.45%	8.67%	11.24%

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Portfolio	3.94%	-3.13%	8.92%	23.84%	12.96%	14.70%	31.06%	9.24%	3.66%
SPY	-37.33%	26.97%	15.59%	4.27%	15.40%	28.55%	13.18%	2.44%	-3.35%
Salient Index	-17.42%	9.38%	11.42%	8.74%	14.36%	0.69%	8.17%	-12.22%	0.94%

Table 6. Multifactor portfolio performance stats

Results (annualized on daily data)			
Parameter	**Portfolio**	**SPY**	**Salient index**
Mean return	15.54%	6.39%	6.26%
St dev	9.23%	19.91%	8.67%
Sharpe	1.47	0.22	0.49
Drawdown	15.84%	54.63%	30.84%
Turnover	619%	0.00%	N/A
Leverage max	3.99	1.00	N/A
Leverage average	2.65	1.00	N/A
Leverage min	0.94	1.00	N/A
Correlation to SPY	0.25	1.00	0.42
Beta	0.11	1.00	0.18
Max one year realized volatility	15%	47%	N/A
Min one year realized volatility	5%	9%	N/A
End value (100 at beginning)	1,356	214	276

Table 7 presents factor correlation table. The average correlation is only 0.03 during overall tested period from 31 Dec 1998 to 26 Feb 2016 in comparison to traditional portfolio where the parameter is 0.14.

Table 7. Factor portfolios correlation table

Tested period: 31 Dec 1998 - 26 Feb 2016

Correlation	TLT	BND	BNDX	Dynamic	Estimates	RWM	Low vol	Small cap	Momentum	Value	Quality	Smom	Sval	Squal
TLT	1.00	0.71	0.64	(0.34)	(0.33)	0.28	(0.27)	(0.28)	(0.31)	(0.35)	(0.33)	0.33	0.32	0.31
BND	0.71	1.00	0.56	(0.19)	(0.18)	0.19	(0.09)	(0.16)	(0.17)	(0.17)	(0.18)	0.21	0.19	0.18
BNDX	0.64	0.56	1.00	(0.24)	(0.21)	0.20	(0.13)	(0.19)	(0.21)	(0.22)	(0.21)	0.26	0.24	0.24
Dynamic	(0.34)	(0.19)	(0.24)	1.00	0.89	(0.75)	0.73	0.74	0.90	0.91	0.90	(0.75)	(0.79)	(0.72)
Estimates	(0.33)	(0.18)	(0.21)	0.89	1.00	(0.76)	0.81	0.67	0.94	0.92	0.95	(0.69)	(0.74)	(0.67)
RWM	0.28	0.19	0.20	(0.75)	(0.76)	1.00	(0.66)	(0.45)	(0.74)	(0.74)	(0.76)	0.73	0.75	0.68
Low vol	(0.27)	(0.09)	(0.13)	0.73	0.81	(0.66)	1.00	0.53	0.77	0.81	0.81	(0.60)	(0.63)	(0.58)
Small cap	(0.28)	(0.16)	(0.19)	0.74	0.67	(0.45)	0.53	1.00	0.67	0.68	0.68	(0.53)	(0.56)	(0.51)
Momentum	(0.31)	(0.17)	(0.21)	0.90	0.94	(0.74)	0.77	0.67	1.00	0.90	0.93	(0.67)	(0.74)	(0.65)
Value	(0.35)	(0.17)	(0.22)	0.91	0.92	(0.74)	0.81	0.68	0.90	1.00	0.93	(0.72)	(0.75)	(0.69)
Quality	(0.33)	(0.18)	(0.21)	0.90	0.95	(0.76)	0.81	0.68	0.93	0.93	1.00	(0.70)	(0.75)	(0.68)
Smom	0.33	0.21	0.26	(0.75)	(0.69)	0.73	(0.60)	(0.53)	(0.67)	(0.72)	(0.70)	1.00	0.90	0.89
Sval	0.32	0.19	0.24	(0.79)	(0.74)	0.75	(0.63)	(0.56)	(0.74)	(0.75)	(0.75)	0.90	1.00	0.88
Squal	0.31	0.18	0.24	(0.72)	(0.67)	0.68	(0.58)	(0.51)	(0.65)	(0.69)	(0.68)	0.89	0.88	1.00

Summary of factor investing.

Key Highlights:

- Underlying instruments: risk factors instead of asset classes
- Forecasting models: static exposure to risk classes or factor timing models
- Asset allocation model: full mean-variance optimization (MVO) or risk parity approach (naive or equal risk)
- Typically required: leverage, long/short exposure, derivatives

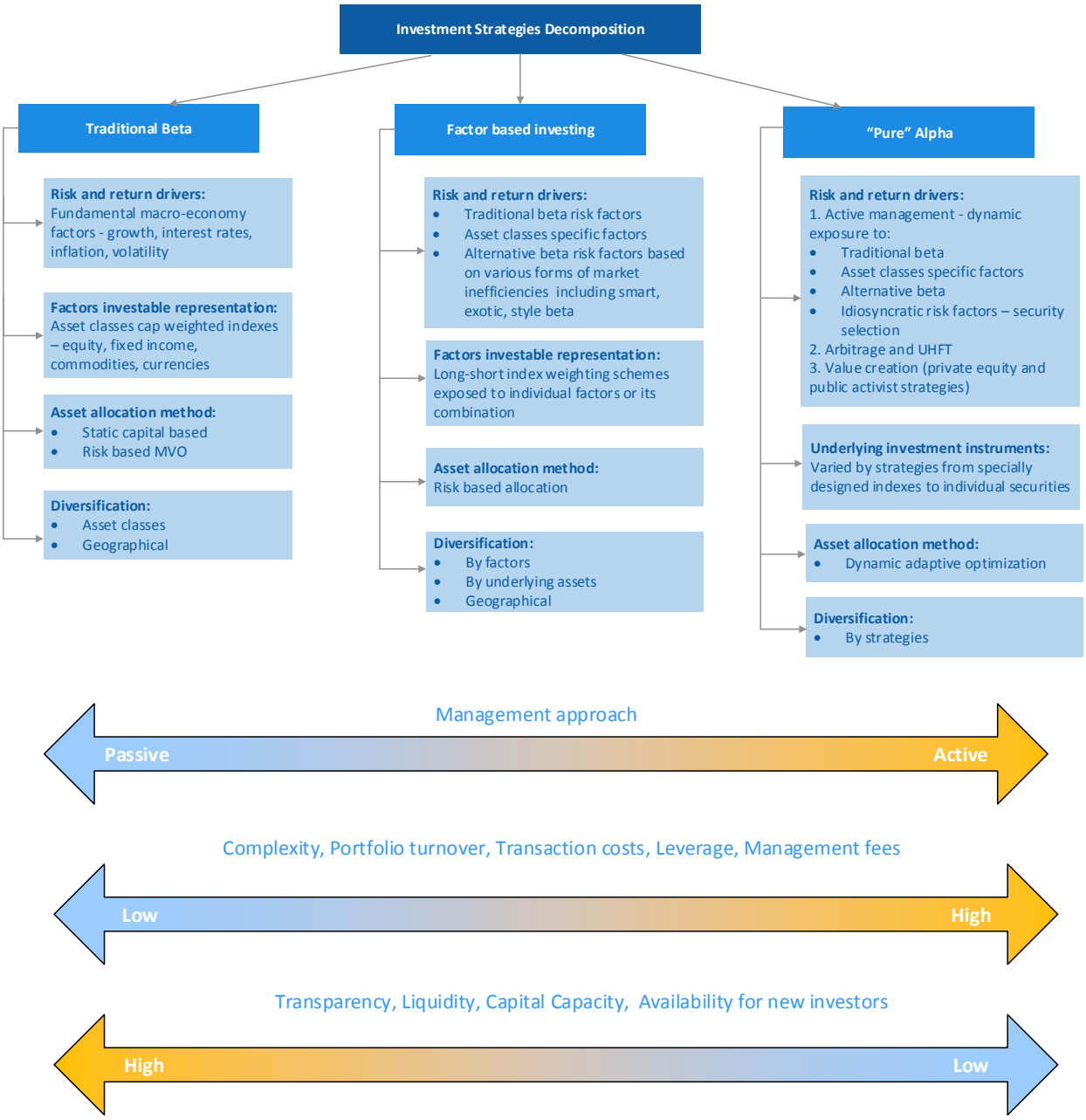
Advantages:

- An increased ability to understand, predict and explain the drivers of performance over different time horizons and regimes
- Much lower factors pairwise correlation (MVO is more robust)
- An ability to unwind from undesired risk exposure
- Return distribution is more balanced – no large drawdowns
- The ability to use casual/Bayesian approaches in forecasting models
- As a result much higher return/risk ratio (Sharpe) relative to traditional approach

Disadvantages:

- The need to determine risk factor universe
- Factors investability, liquidity and practical capturing challenges
- Required use of derivatives, short positions and high leverage – not appropriate for all investors
- High turnover (frequent rebalancing), leverage and transaction costs for “pure” factors and dynamic allocation
- Low cost on ready to use low turnover long only indexes, high cost for “pure” market neutral factors
- Secondary factor tilts control

IV. Traditional beta – Alternative Beta - Alpha



Risk/return decomposition = cash interest + beta + alternative beta + alpha + stochastic component

The depicted scheme represents three categories of risk and return drivers of investment portfolio in capital markets. The first block is traditional beta, the second is factor based investing and the final most valuable category is "pure" alpha (alpha comparing to static beta and alternative beta) derived from the following sources:

1. Idiosyncratic risk exposure: security selection based on superior information or analytical skill
2. Market and factor timing models (volatility, return and correlation forecasts)
3. Portfolio adaptive dynamic allocation (optimization techniques)
4. Event-driven strategies, arbitrage and ultra high frequency trading
5. Value creation (top private equity funds or activist strategies in public markets)

The cost, complexity and the rate of asset management activity rise significantly from static beta to pure alpha strategies.
The main idea of active management (according to the Fundamental Law of Active Management) is to determine and combine as many independent alpha sources as possible in order to generate high and stable investment results uncorrelated to traditional assets.

On *figure 21* beta timing model presented as example of "pure" alpha factor timing strategy. The model consists from nine macro-economy and equity related parameters and statistics on monthly basis from the following sources: Federal Reserve Bank of St. Louis, Recessionalert.com, Portfolio123.com. The purposes of this model is to predict large recessionary market drawdowns and spikes of volatilities. The strategy simply buys S&P futures (dividends excluded) when the model predicts positive risk premia and shorts it when economy recession is coming. The model correlation to future six month realized S&P return (prediction power) is only 0.5 but it is enough to avoid large drawdowns and significantly increase performance in comparison to buy and hold strategy.

Figure 21. Equity market beta timing model example

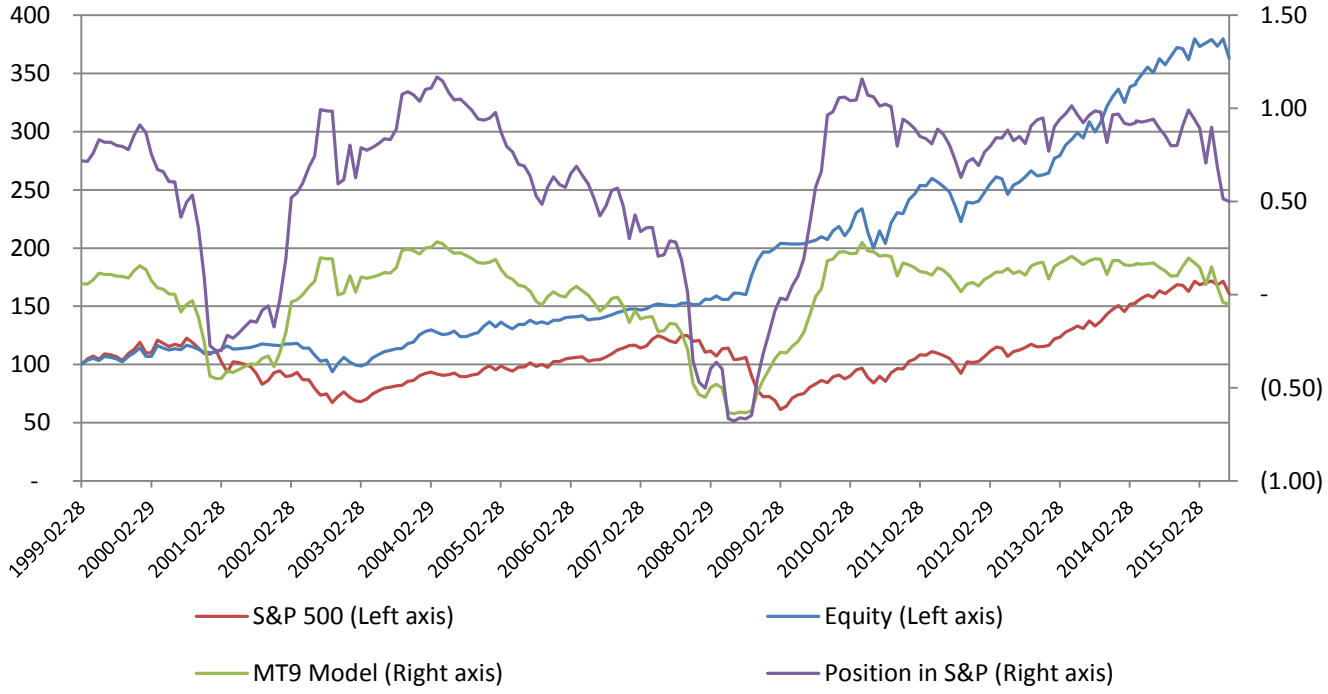


Figure 22. Performance stats for market timing strategy

Parameter	MT9 System	S&P500
Mean return	8.35%	4.02%
St dev	10.16%	15.05%
Return/Risk	0.82	0.27
Turnover	84.81%	0.00%
Drawdown	-20.61%	-50.84%

V. What do we offer?

Products and services:

- Determined global investable factor universe database that includes popular traditional, alternative and not widely known pure alpha risk/return factors
- Effective factor exposure via ranking and optimization approach which is superior to popular alternative indexes weighting methods
- Robust factor timing models
- Robust portfolio optimization schemes
- Goal seeking theme investing (theme is a combination of factors with a special purpose)
- Diversified pure alpha multi-strategy portfolios at low cost

The key products characteristics and advantages are the following:

- Effective implementation - based on robust logic and economic intuition, automatically captures three "pure" alpha sources and exploits the all three factor level premiums most possibly efficient, gives us an ability to operate by "pure" factors and individual securities.
- System return distribution is balanced – there is no fat left tails and large drawdowns even during crisis periods.
- Limited exposure to traditional beta risk sources (traditional beta is the one factor among many others) – low correlation to the global growth risk.
- Leads to outstanding results once launched onto the global asset universe which is confirmed by 16 years back-tests and out-of-sample live results.

Contacts and further steps.

Full presentation, strategies descriptions, performance and other details are available upon personal request. For potential cooperation contact Advantex LLC authorized person - Yury Polyakov.

Phone: +7 (495) 721-47-74
Email: info@advantex.ru

Author's personal contacts:

Phone: +7 (906) 706-99-90
Email: yupolv@gmail.com

VI. Appendix

Alpha (α)
A measure of performance. Active risk adjusted abnormal return relative to a benchmark.

Benchmark
A standard against which the performance of a portfolio can be measured.

Beta (β)
A measure of the sensitivity of an asset to movements in the market; thus, a measure of the asset's non-diversifiable or systematic risk.

Correlation
A statistical term giving the strength of a linear relationship between two random variables. It is a pure number, ranging from –1 to +1. A correlation of +1 indicates perfect positive linear relationship; –1, perfect negative linear relationship; 0, no linear relationship.

Covariance matrix
The square matrix containing the variances (along the diagonal) and covariance (off-diagonal) of all the common factors in a risk model. It is a key component in the forecasting of risk measures.

Descriptor
A fundamental or market-related data item that is used as a fundamental building block of risk index or style factors in a Barra equity risk model. Most style factors or risk indices are comprised of several descriptors combined using proprietary formulas. For example, a volatility risk index, which distinguishes high volatility assets from low volatility assets, might consist of several descriptors based on short-term volatility, long-term volatility, systematic volatility, and residual volatility, and so on.

Diversification
The spreading of risk by investing in a number of different assets whose returns are not perfectly positively correlated. Since the returns are not perfectly correlated, losses of any one asset tend to be offset by gains on other assets. In this manner, the risk of a portfolio may well be less than the average risk of its constituent assets.

Downside Deviation
The standard deviation for all negative returns in your portfolio in the specific time period.

EMH
The efficient market hypothesis (EMH) is an investment theory that states it is impossible to "beat the market" because stock market efficiency causes existing share prices to always incorporate and reflect all relevant information.
There are three variants of the hypothesis: "weak", "semi-strong", and "strong" form. The weak form of the EMH claims that prices on traded assets (e.g., stocks, bonds, or property) already reflect all past publicly available information. The semi-strong form of the EMH claims both that prices reflect all publicly available information and that prices instantly change to reflect new public information. The strong form of the EMH additionally claims that prices instantly reflect even hidden "insider" information.

Excess return
Return in excess of the risk-free rate. The excess return is computed by subtracting the promised risk-free rate from a security's return.

Exposure
A term used to quantify the magnitude of an asset's (or portfolio's) sensitivity to factors.

Factor
Risk factor is a casual driver of asset returns which has risk, return and relationship characteristics with other risk factors. All risk factors have a degree of uncertainty or risk associated with them, and a corresponding risk premium which may be positive or negative. Examples of equity factors are: size, value, growth, and earnings variation. Examples of fixed-income factors are: shift, twist, and butterfly.

Factor return
The return attributable to a particular common factor.

Liquidity
In its modern usage, the liquidity of an asset is the extent to which it can be readily converted into cash without paying a large spread or moving the market.

Max Drawdown
The largest cumulative percentage decline in the Net Asset Value of your portfolio from the highest or peak value to the lowest or trough value after the peak.

Mean Return
The average time weighted return of a portfolio for a specified time period.

Momentum
Rate of acceleration of an economic, price, or volume movement. An economy with strong growth that is likely to continue is said to have a lot of momentum. In the stock market, technical analysts study stock momentum by charting price and volume trends.

Optimization
The best solution among all the solutions available for consideration. Constraints on the investment problem limit the region of solutions that are considered, and the objective function for the problem, by capturing the investor's goals correctly, provides a criterion for comparing solutions to find the better ones. The optimal solution is the solution among those admissible for consideration that has the highest value of the objective function. The first-order conditions for optimality express the trade-offs between alternative portfolio characteristics to provide the optimum solution.

Outlier
A data observation that is very different from other observations.

Peak-to-Valley
The time period during which the Max Drawdown (largest cumulative percentage decline in the NAV) occurred.

Risk
The uncertainty of investment outcomes. Technically, risk defines all uncertainty about the mean outcome, including both upside and downside possibilities. Studies of investment return have shown very consistently that when returns are centered about their expected value, there is little difference between the extent of upside and downside variability relative to that value. Thus a measure of total variability in both directions is typically used to summarize risk. The more intuitive concept for risk measurement is the standard deviation of the distribution, a natural measure of the spread. Variance, the square of the standard deviation, must be used in comparing independent elements of risk. On the other hand, when the shapes of the upper and lower tails of the probability distribution are different—as they are in the case of catastrophic default risk, or whenever an option is present—it may be necessary to take these into account and analyze the risky distribution more completely. A more complete analysis can be accomplished by taking into account not only the spread of the distribution (standard deviation or variance) but also the asymmetry or skewness of the distribution.

Standard deviation
A statistical term which measures the spread of variability of a probability distribution. It is the square root of variance. Standard deviation is widely used as a measure of risk or volatility of portfolio investments. A higher standard deviation indicates a product with more risk. A product's portfolio is expected to differ positively or negatively from the mean return by no more than the standard deviation amount for approximately 68% of its cycle.

Sortino Ratio
The ratio measures the risk adjusted return of an investment. The ratio penalizes only those returns that fall below the required rate of return.

Sharpe Ratio
A ratio that measures the excess return per unit of risk. The ratio is used to characterize how well the return compensates an investor for the risk taken.

Trading costs
Costs of buying and selling securities and borrowin g. Trading costs include commissions, slippage, and the bid/ask spread.

Universe
The list of all assets eligible for consideration for inclusion in a portfolio.

Value at risk (VaR)
A measure that characterizes the potential loss in currency units in a given time period for a given probability level. For example, a VaR of –1,000,000 at the 5% probability level indicates there is a 5% probability one would lose up to 1,000,000 in the coming year.

Variance
A measure of the variability of variables around the mean. Variance is defined as the expected squared deviation of the random variable from its mean—that is, the average squared distance between the mean value and the actually observed value of the random variable. When a portfolio includes several independent elements of risk, the variance of the total arises as a summation of the variances of the separate components.

Volatility
A measure of a share's propensity to go up and down in price. A volatile share is one which has a tendency to move drastically across a broad share price range. Mathematically, this is expressed as the standard deviation from the average performance.

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